

6.2

Properties of Parallelograms

146

- Goals**
- Use some properties of parallelograms.
 - Use properties of parallelograms in real-life situations.

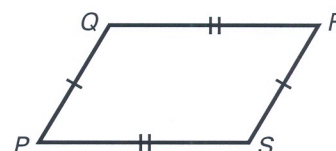
VOCABULARY

Parallelogram

THEOREM 6.2

If a quadrilateral is a parallelogram, then its **opposite sides** are congruent.

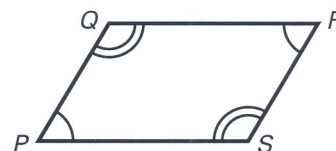
$$\underline{\hspace{1cm}} \cong \overline{RS} \text{ and } \overline{SP} \cong \underline{\hspace{1cm}}$$



THEOREM 6.3

If a quadrilateral is a parallelogram, then its **opposite angles** are congruent.

$$\angle P \cong \angle \underline{\hspace{1cm}} \text{ and } \angle \underline{\hspace{1cm}} \cong \angle S$$



THEOREM 6.4

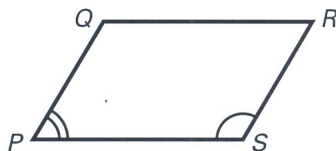
If a quadrilateral is a parallelogram, then its **consecutive angles** are supplementary.

$$m\angle P + m\angle Q = \underline{\hspace{1cm}}^\circ,$$

$$m\angle Q + m\angle R = \underline{\hspace{1cm}}^\circ,$$

$$m\angle R + m\angle S = \underline{\hspace{1cm}}^\circ,$$

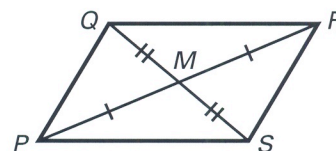
$$m\angle S + m\angle P = \underline{\hspace{1cm}}^\circ$$



THEOREM 6.5

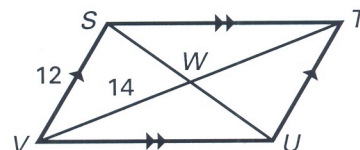
If a quadrilateral is a parallelogram, then its diagonals **bisect** each other.

$$\overline{QM} \cong \underline{\hspace{1cm}} \text{ and } \underline{\hspace{1cm}} \cong \overline{RM}$$



Example 1 Using Properties of Parallelograms

$STUV$ is a parallelogram. Find the unknown length.



- a. TU b. WT

Solution

a. $TU = \underline{\hspace{2cm}}$ Opposite sides of a \square are \cong .

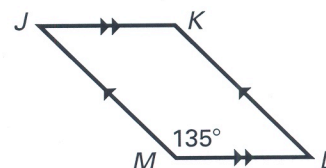
$TU = \underline{\hspace{2cm}}$ Substitute $\underline{\hspace{1cm}}$ for $\underline{\hspace{1cm}}$.

b. $WT = \underline{\hspace{2cm}}$ Diagonals of a \square bisect each other.

$WT = \underline{\hspace{2cm}}$ Substitute $\underline{\hspace{1cm}}$ for $\underline{\hspace{1cm}}$.

Example 2 Using Properties of Parallelograms

$JKLM$ is a parallelogram. Find $m\angle L$.



Solution

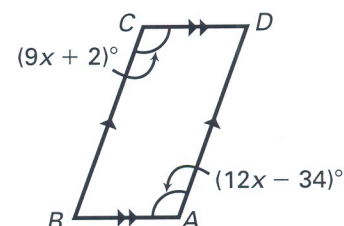
$m\angle L + m\angle \underline{\hspace{1cm}} = \underline{\hspace{1cm}}^\circ$ Consecutive angles of a \square are supplementary.

$m\angle L + \underline{\hspace{1cm}}^\circ = \underline{\hspace{1cm}}^\circ$ Substitute $\underline{\hspace{1cm}}^\circ$ for $m\angle \underline{\hspace{1cm}}$.

$m\angle L = \underline{\hspace{1cm}}^\circ$ Subtract $\underline{\hspace{1cm}}^\circ$ from each side.

Example 3 Using Algebra with Parallelograms

$ABCD$ is a parallelogram. Find the value of x .



Solution

$$m\angle \underline{\hspace{1cm}} = m\angle \underline{\hspace{1cm}}$$

$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}} - \underline{\hspace{1cm}}$$

$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$\underline{\hspace{1cm}} = \underline{\hspace{1cm}} x$$

$$\underline{\hspace{1cm}} = x$$

Opposite angles of a \square are \cong .

Substitute.

Add $\underline{\hspace{1cm}}$ to each side.

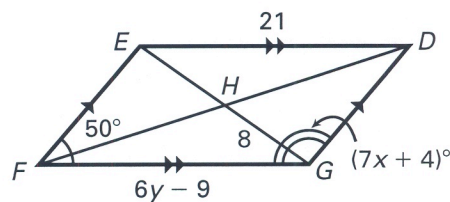
Subtract $\underline{\hspace{1cm}}$ from each side.

Divide each side by $\underline{\hspace{1cm}}$.

✓ **Checkpoint** Find the measure or value in parallelogram $DEFG$. Explain your reasoning.

$m\angle EFG = 50^\circ$

1. Find ~~$m\angle D$~~ . $m\angle EDG$



2. Find EH .

3. Find the value of y in the parallelogram above.

4. Find the value of x in the parallelogram above.

Hint # 4
Use $\angle FED \cong \angle DGF$
What is $m\angle FED$?

Example 4 Using Parallelograms in Real Life

Gemstones A gemstone is cut so that one of its facets has four sides. The measures of the consecutive angles in the facet are 45° , 135° , 135° , and 45° . Is the facet a parallelogram? Explain.

Solution

The facet is not a parallelogram. Here are two reasons why.

- The opposite angles _____.
- The sums of the measures of the consecutive angles are _____ $^\circ$, _____ $^\circ$, _____ $^\circ$, and _____ $^\circ$. If the facet were a parallelogram, then all pairs of consecutive angles would be _____.

Study Guide

6.1/6.2 Blue

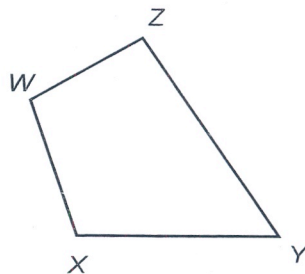
Quadrilaterals

A **quadrilateral** is a closed geometric figure with four sides and four vertices. Any two sides, vertices, or angles of a quadrilateral are said to be either **consecutive** or **opposite**.

A segment joining any two nonconsecutive vertices in a quadrilateral is called a **diagonal**.

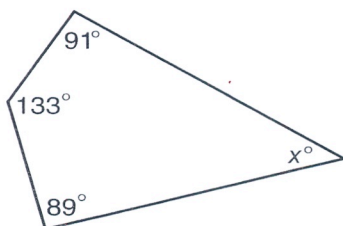
Refer to quadrilateral WXYZ for Exercises 1–5.

1. Name all pairs of opposite sides.
2. Name all pairs of consecutive angles.
3. Name the diagonals.
4. Name all pairs of consecutive vertices.
5. Name all pairs of opposite angles.

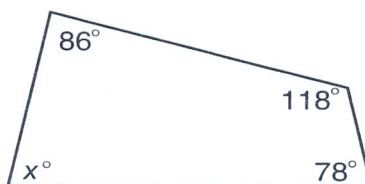


Find the missing measure(s) in each figure.

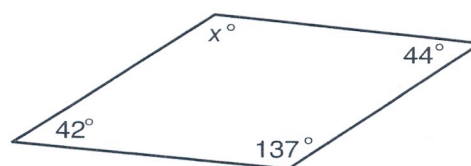
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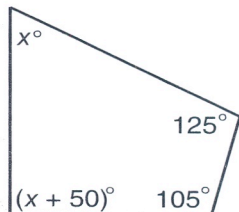
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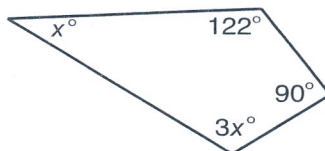
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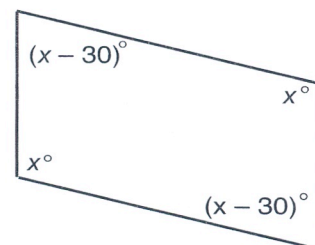
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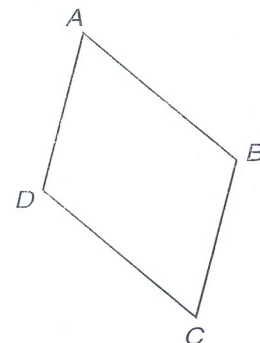
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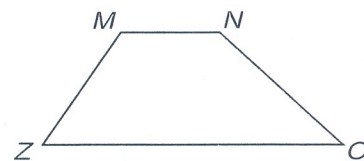
Skills Practice

6.1/6.2 Blue

Quadrilaterals

Refer to quadrilaterals $ABCD$ and $MNOZ$.

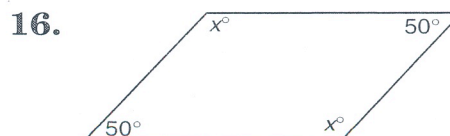
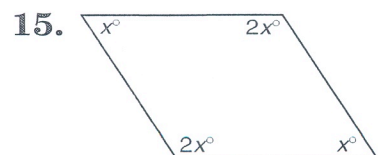
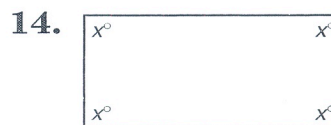
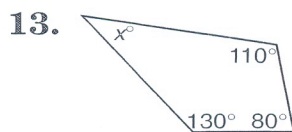
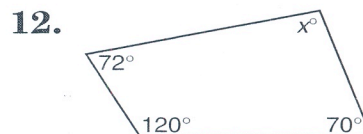
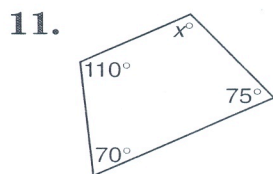
Exercises 1 - 5



Exercises 6 - 10

1. Name the side opposite \overline{BC} .
2. Name a side that is consecutive with \overline{AB} .
3. Name a pair of consecutive vertices in $ABCD$.
4. Name the vertex opposite B .
5. Name the two diagonals in $ABCD$.
6. Name a pair of consecutive angles in $MNOZ$.
7. Name the vertex opposite O .
8. Name the side opposite \overline{MZ} .
9. Name a side that is consecutive with \overline{MN} .
10. Name the diagonals in $MNOZ$.

Find the missing measure(s) in each figure.



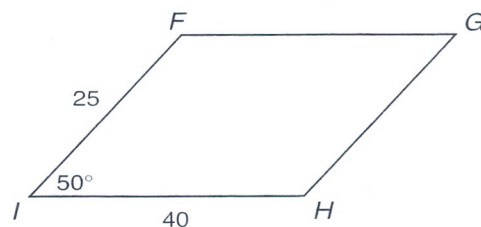
Find each measure.

1. $m\angle H$

2. $m\angle G$

3. GH

4. FG



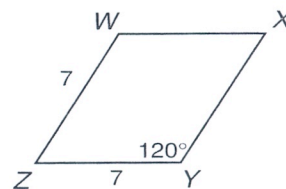
Find each measure.

5. $m\angle Z$

6. $m\angle W$

7. XY

8. WX



In the figure, $TQ = 42$ and $SA = 14$. Find each measure.

9. TA

10. $m\angle QST$

11. SR

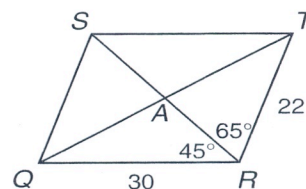
12. $m\angle STR$

13. SQ

14. ST

15. AQ

16. AR



17. In a parallelogram, the measure of one side is 38. Find the measure of the opposite side.

18. The measure of one angle of a parallelogram is 45. Determine the measures of the other three angles.

Study Guide

6.2 Blue

149

Parallelograms

A special kind of quadrilateral in which both pairs of opposite sides are parallel is called a **parallelogram**.

The following theorems all concern parallelograms.

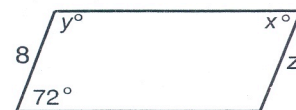
- Opposite sides of a parallelogram are congruent.
- Opposite angles of a parallelogram are congruent.
- Consecutive angles of a parallelogram are supplementary.
- The diagonals of a parallelogram bisect each other.

Example: If the quadrilateral in the figure is a parallelogram, find the values of x , y , and z .

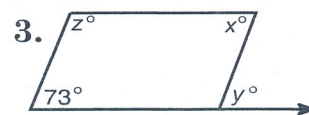
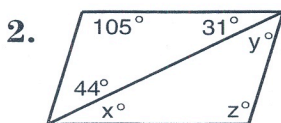
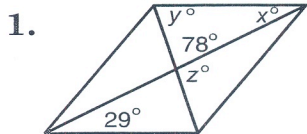
Since opposite angles of a parallelogram are congruent, $x = 72$.

Since consecutive angles of a parallelogram are supplementary, $y + 72 = 180$. Therefore, $y = 108$.

Since opposite sides of a parallelogram are congruent, $z = 8$.



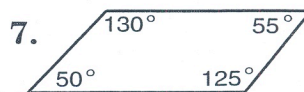
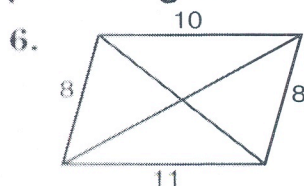
If each quadrilateral is a parallelogram, find the values of x , y , and z .



4. In parallelogram $ABCD$, $m\angle A = 3x$ and $m\angle B = 4x + 40$. Find the measure of angles A , B , C , and D .

5. In parallelogram $RSTV$, diagonals \overline{RT} and \overline{VS} intersect at Q . If $RQ = 5x + 1$ and $QT = 3x + 15$, find QT .

Explain why it is impossible for each figure to be a parallelogram.

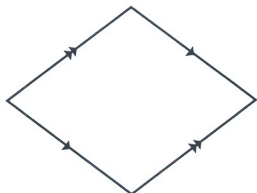


Practice A

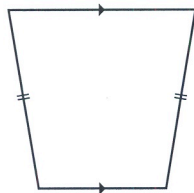
For use with pages 330–337

Decide whether the figure is a parallelogram. If it is not, explain why not.

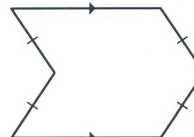
1.



2.



3.



Use the diagram of parallelogram $MNOP$ at the right. Complete the statement, and give a reason for your answer.

4. $\overline{MN} \cong$?

5. $\overline{MN} \parallel$?

6. $\overline{ON} \cong$?

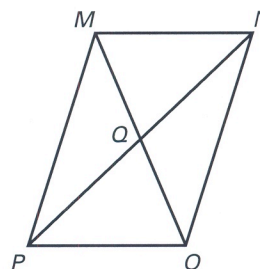
7. $\angle MPO \cong$?

8. $\overline{PQ} \cong$?

8. $\overline{QM} \cong$?

10. $\angle MQN \cong$?

11. $\angle NPO \cong$?



Find the measure in the parallelogram $HJKI$. Explain your reasoning.

12. HI

13. KH

14. GH

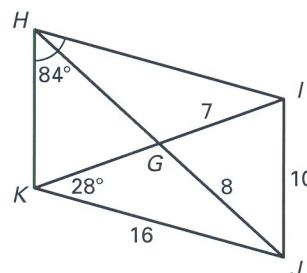
15. HJ

16. $m\angle KIH$

17. $m\angle JIH$

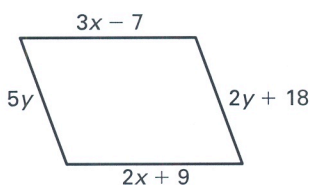
18. $m\angle KJI$

19. $m\angle HKI$

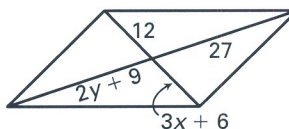


Find the value of each variable in the parallelogram.

20.



21.



Complete the flow-proof at the right.

22. Given: $\square ABCD$

Prove: $\triangle ABD \cong \triangle CDB$

